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Résumé. Nous examinons un problème multi-périodes d'expansion de la capacité dans un réseau d'accès local de télécommunications ayant une topologie d'arbre. Cette expansion est réalisée en ajoutant des concentrateurs aux sommets et des câbles sur les liens du réseau. Le but est de trouver une installation de moindre coût qui satisfasse la demande. Une approche de résolution heuristique est proposée où des décisions d'installation locales à chaque sommet sont propagées dans tout le réseau. Cette information est ensuite utilisée afin d'ajuster des prix qui guident la prise de décision d'une itération à l'autre, jusqu'à ce qu'un équilibre soit atteint. Des résultats numériques sont présentés pour des instances ayant diverses structures de coût et de demande.

Mots clés: Télécommunications, réseau d'accès local, expansion de la capacité, multi-périodes, heuristique.

Abstract. In this paper, we examine a multi-period capacity expansion problem for a local access telecommunications network with a tree topology. Capacity expansion is realized through the installation of concentrators at the nodes and cables on the links of the network. Clearly, the installation of concentrators reduces the need for additional cables, and conversely. The goal is to find the least cost alternative to satisfy the demand. A heuristic approach is proposed to solve this problem, where local installation decisions at each node are propagated in the network. This information is then used to adjust prices that guide the decision process from one iteration to the next, until a fixed point is reached. Numerical results are reported on problem instances based on different cost and demand structures.

Keywords: Telecommunications, local access network, capacity expansion, multi-period, heuristic.

1 Introduction

There are many variants of network capacity expansion problems depending on the application considered and the type of network under study. In the telecommunications industry, a network is often composed of a backbone network, for the transfer of large volumes of data, and local access networks that connect terminals to an access node of the backbone network. Starting with the pioneering work in [7, 8, 16], the study of network capacity expansion problems has continually "expanded"' over the years. But, researchers in this field have mostly focused on backbone networks, while the local access networks that feed them have been relatively neglected. Furthermore, most network planning models deal with a single time period, as opposed to more realistic multi-period expansion strategies.

In this paper, we consider the problem of finding a least-cost multi-period capacity expansion strategy for a local access network with a tree topology, where the expansion can be done through the installation of flow compression devices, known as concentrators, at the nodes and cables on the links of the networks. Recent papers on capacity expansion for backbone networks (including, in some cases, network topology considerations) may be found in [3, 4, 15], with multi-period expansion strategies reported in [4, 10]. With regard to local access networks, most studies deal with topology design [5, 12, 13, 14]. However, in [1, 2], the authors address the capacity expansion problem via an optimization-based algorithm, where a decomposition method combines Lagrangean relaxation with a dynamic programming algorithm. The authors consider both the installation of concentrators and cables to accommodate the demand, but over a single time period. An exact dynamic programming algorithm for solving a similar problem is also reported in [11]. To the best of our knowledge, the work reported in [6] is the only one that addresses the multi-period capacity expansion problem for local access networks. In this work, a non linear mixed-integer programming model is linearized and solved through Lagrangian relaxation. A small problem instance on a tree with 20 nodes and 4 time periods is solved with this approach. The heuristic reported here can handle much larger problem instances, as shown in Section 4. Our model is also closer to reality by considering modular capacities for the cables and cost structures with economies of scale (in [6], the capacity is continuous and can be precisely adjusted to the flow; furthermore, the cost increases linearly with capacity).

The remainder of the paper is as follows. In Section 2, a mathematical programming formulation of the problem is proposed. The heuristic problem-solving approach is then presented in Section 3. Computational experiments on problem instances based on different cost and demand structures are reported in Section 4. Finally, concluding remarks follow in Section 5.

2 Problem Formulation

We assume a local access network $G = (V, A)$ where V is the set of nodes and A is the set of arcs. The network exhibits a tree topology (by removing the orientation on the arcs) with only one node directly connected to the access node. An example is presented in Figure 1. In the figure, $Down(b) = Down(c) = a$ is the immediate downstream node of b and c, while $Down(d) = Down(e) = b$ is the immediate downstream node of d and e. Conversely, $Up(a) = \{b, c\}$ and $Up(b) = \{d, e\}$ are the sets of immediate upstream nodes of a and b, respectively.

The problem is to plan the installation of concentrators at the nodes (excluding the access node) and cables on the arcs over a number of time periods to minimize the installation costs. A variant of this problem is considered here, where the following assumptions hold:

- at each node, only one concentrator can be installed over the entire planning horizon;
- once installed at a node, a concentrator remains there until the end of the planning horizon (i.e., it cannot be removed).

- on each arc, cables can only be added to existing ones over the planning horizon;
- the costs associated with concentrators and cables decrease over time, due to obsolescence; thus, it is not advantageous to install equipments too early.

In the mathematical formulation presented below, the following notation is used:

- L is the set of concentrator types, where a type is characterized by its compression factor ϕ_l, $l \in L$, $\phi_l \leq 1$ (a value of 1 means that there is no flow compression),
- Q is the set of cable types, where a type is characterized by its bandwidth capacity β_q, $q \in Q$,
- $T = \{1, 2, ..., |T|\}$ is the set of time periods of the planning horizon,
- D_{it} is the demand at node i at period t, $i \in V$, $t \in T$,
- c_{qijt} is the cost associated with the installation of a cable of type q on arc (i,j) at period t, $q \in Q$, $(i,j) \in A$, $t \in T$,
- f_{lit} is the cost associated with the installation of a concentrator of type l at node i at period t, $l \in L$, $i \in V$, $t \in T$,

The problem can now be formulated as follows:

$$F = \min \sum_{t \in T} \sum_{l \in L} \sum_{i \in V} f_{lit} Z_{lit} + \sum_{t \in T} \sum_{q \in Q} \sum_{(i,j) \in A} c_{qijt} X_{qijt}, \tag{1}$$

subject to

$$\left(\sum_{h \in Up(i)} Y_{hit} + D_{it} \right) \left(\sum_{t' \in T, t' \leq t} \sum_{l \in L} (\phi_l - \Phi_i) Z_{lit'} + \Phi_i \right) = Y_{ijt}, \quad i \in V, j = Down(i), t \in T, \tag{2}$$

$$Y_{ijt} \leq \sum_{t' \in T, t' \leq t} \sum_{q \in Q} \beta_q X_{qijt'}, \quad (i,j) \in A, t \in T, \tag{3}$$

$$\sum_{t \in T} \sum_{l \in L} Z_{lit} \leq 1, \quad i \in V, \tag{4}$$

$$X_{qijt} \geq 0 \text{ and integer}, \quad q \in Q, (i,j) \in A, t \in T, \tag{5}$$

$$Y_{ijt} \geq 0, \quad (i,j) \in A, t \in T, \tag{6}$$

$$Z_{lit} \in \{0,1\}, \quad l \in L, i \in V, t \in T, \tag{7}$$

where

- X_{qijt} is the number of cables of type q installed on arc (i,j) at period t, $q \in Q$, $(i,j) \in A$, $t \in T$,
- Z_{lit} indicates if a concentrator of type l is installed at node i at period t, $l \in L$, $i \in V$, $t \in T$,
- Y_{ijt} is the flow on arc (i,j) at period t, $(i,j) \in A$, $t \in T$.

The flow conservation constraints (2) establish a relation between the flow that enters a node (incoming flow plus local demand) and the flow that goes out, possibly after compression. Note that Φ_i is the initial concentration level at node i, which is usually set to 1 (no flow compression). Constraints (3) hold when the capacity installed on each arc is sufficient to handle the corresponding flow. Constraints (4) require that at most one concentrator be installed at a given node over the entire planning horizon. Note that the Z_{lit} variables can be seen as transition variables, as they indicate if a transition from the initial state takes place or not at a particular node (i.e., if a concentrator is installed at some period or not).

3 Problem-Solving Approach

The algorithm devised for solving the multi-period capacity expansion problem of Section 2 is an iterative heuristic where, at each iteration, new installation decisions are taken at each node of the network based on prices that are modified from one iteration to the next. The algorithm stops when an equilibrium point is attained (i.e., when the prices do not change anymore), or when a given number of iterations is reached.

A solution to the problem is made of all concentrators and cables that are installed at the nodes and arcs of the network to satisfy the demand, at each time period. In the following subsections, the iterative heuristic is described where, at each iteration, a downstream pass for generating installation decisions is followed by an upstream pass for adjusting the prices.

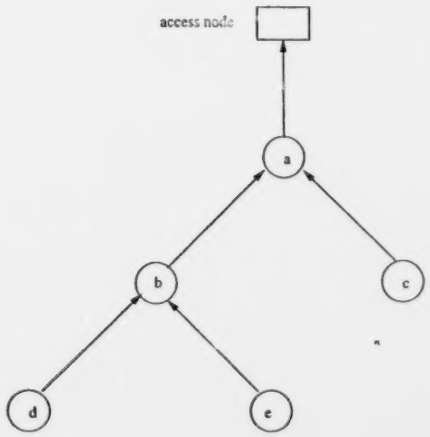

Figure 1: Local Access Network

3.1 Downstream pass

At each iteration k, local installation decisions are taken at every node of the network. This is done by starting at the leaves and proceeding downstream until the node directly connected to the access node is reached. These local decisions relate to the (possible) installation of a concentrator at a node and cables on the outgoing arc to accommodate the flow. As it is necessary to know the amount of incoming flow to take appropriate installation decisions, a node can be processed only when all upstream nodes have already been processed.

Assuming that the incoming flow at node i at each time period is known (i.e., installation decisions have already been taken for all upstream nodes), decisions about the installation of a concentrator at the node and cables on the outgoing arc, over the planning horizon, are taken to minimize:

$$F_i^k = \sum_{t \in T} \sum_{l \in L} f_{lit} Z_{lit} + \sum_{t \in T} \sum_{q \in Q} c_{qijt} X_{qijt} + \sum_{t \in T} p_{jt}^k Y_{ijt}, \; i \in V,$$

where $Down(i) = j$ is the immediate downstream node. The first component of F_i^k is the installation cost of a concentrator at a given time period, if any (recall that at most one Z_{lit} variable can be set to 1); the second component corresponds to the installation cost of all cables that are necessary to accommodate

the flow Y_{ijt} on arc (i,j), $t \in T$; the third component is the total price charged by node j for the flow Y_{ijt} sent on arc (i,j), $t \in T$. Note that the prices p^k_{jt} are initially set to $p^0_{jt} = 0, j \in V, t \in T$, but are modified from one iteration to the next to induce different installation decisions (see subsection 3.2).

We find the minimum of F^k_i by considering, in turn, the installation of each type of concentrator at each period of the planning horizon. With $|T|$ time periods and $|L|$ types of concentrators, there are $|T||L| + 1$ alternatives, if we include the possibility of not installing any concentrator. For each alternative, the cost associated with the concentrator (first component of F^k_i) is known. Furthermore, the outgoing flows $Y_{ijt}, t \in T$ on arc (i,j) are easily calculated. We can thus evaluate the cost of the cables that must be installed to accommodate them (second component of F^k_i) as well as the total price charged by node j over the planning horizon (third component of F^k_i). In the first case, the dynamic programming algorithm reported in [9] is applied to find the least cost combination of cables for each period of the planning horizon. In the second case, the total price charged by node j is easily calculated, given that the price per unit of flow and outgoing flow on arc (i,j) for each time period are known. When all alternative installations of the concentrator have been evaluated, the best one is returned.

The procedure is summarized below for some node i at iteration k, where variable F^k_i is the cost of the current installation and F^{k*}_i is the cost of the best installation. This procedure takes as input the prices per unit of flow $p^k_{jt}, t \in T$ charged to i by its immediate downstream node j, as well as the incoming flows Y_{rit} at node i, $r \in Up(i)$, $t \in T$. Due to the latter, we recall that this procedure is applied at node i only after all upstream nodes have been processed. Steps 1, 2 and 3 consider no concentrator installation. Step 4 considers the remaining alternatives by iterating over each concentrator type and time period.

1. $Y_{ijt} \leftarrow D_{it} + \sum_{r \in Up(i)} Y_{rit}$, $t \in T$.
2. Find the X_{qijt} values, $q \in Q$, $t \in T$, that minimiże the cost of the cables to accommodate the flow Y_{ijt}, $t \in T$.
3. $F^{k*}_i \leftarrow \sum_{t \in T} \sum_{q \in Q} c_{qijt} X_{qijt} + \sum_{t \in T} p^k_{jt} Y_{ijt}$.
4. For $l \in L$ do

   For $t \in T$ do

   {*a concentrator of type l is installed at node i at time period t*}

   4.1 $Y_{ijt'} \leftarrow D_{it'} + \sum_{r \in Up(i)} Y_{rit'}$, $t' \in T$.

   4.2 $Y_{ijt'} \leftarrow \phi_l Y_{ijt'}$, $t' \in T$, $t' \geq t$.

   4.3 Find the $X_{qijt'}$ values, $q \in Q$, $t' \in T$, that minimize the cost of the cables to accommodate the flow $Y_{ijt'}$, $t' \in T$.

   4.4 $F^k_i \leftarrow f_{lit} + \sum_{t' \in T} \sum_{q \in Q} c_{qijt'} X_{qijt'} + \sum_{t' \in T} p^k_{jt'} Y_{ijt'}$.

   4.5 if $F^k_i < F^{k*}_i$ then $F^{k*}_i \leftarrow F^k_i$.
3. Return F^{k*}_i and the corresponding installations.

When every node has been processed in this manner, we have a solution. That is, the installations (concentrator, cables) at every node and arc of the network over the planning horizon are known. We are thus ready for the following upstream pass, which is described in the next subsection.

3.2 Upstream pass

When the downstream pass at iteration k is completed and installation decisions have been taken at every node of the network, an upstream pass takes place to modify the current prices and get the prices

$p_{jt}^{k+1}, j \in V, t \in T$ to be used at the next iteration. This upstream pass starts at the node directly connected to the access node and ends at the leaves. Basically, every node j charges to its immediate upstream nodes $i \in Up(j)$ a price p_{jt} for each unit of flow sent on link (i,j) at period t. The prices per unit of flow at iteration $k+1$ are determined as follows:

$$p_{jt}^{k+1} = \frac{\sum_{l \in L} f_{ljt} Z_{ljt}^{k*} + \sum_{q \in Q} c_{qjrt} X_{qjrt}^{k*} + p_{rt}^{k+1} Y_{jrt}^{k*}}{\sum_{i \in Up(j)} Y_{ijt}^{k*} + D_{jt}}, \; r = Down(j), \; j \in V, \; t \in T.$$

In this equation, the numerator is the cost associated with node j at period t, where Z^{k*}, X^{k*} and Y^{k*} denote the variable values in the solution obtained at the end of the downstream pass. Clearly, as the updated prices per unit of flow at the immediate downstream node r are needed to update the prices at node j, the latter can only be processed when all its downstream nodes have been processed. The denominator is the total flow handled by node j, that is the incoming flow plus the local demand. Thus, node j charges to each immediate upstream node the cost it incurs to process each unit of flow that goes through it. One exception is the access node, which does not charge anything to its unique immediate upstream node.

An example, based on a single time period $t = 1$, is provided in Figure 2 at some iteration k. Here, the downstream pass has just been completed and we know what the network installations are, as well as the flow on each arc (c.f., labels associated with full arrows). Now, we consider node a, which is directly connected to the access node. The flow going through this node is equal to the incoming flow from nodes b and c, $10 + 20 = 30$, plus a local demand of 20, for a total of 50. As the outgoing flow is only 20, there is clearly a concentrator at this node. Each dashed arrow between two nodes y and x is labeled with the price charged by y to x, namely the price per unit of flow times the flow on arc (x,y). If we assume that the equipment costs (concentrator, cables) installed at node a is 100, then $p_{a1}^{k+1} = \frac{100}{10+20+20} = 2$ per unit of flow. Node a will thus charge a price of $10 * 2 = 20$ to node b and $20 * 2 = 40$ to node c at iteration $k+1$. A price of $20 * 2 = 40$ is also associated with the local demand at node a. If we sum up these three values, we obtain $20 + 40 + 40 = 100$, the total cost incurred by node a to process its flow. When this calculation is done, we can proceed with nodes $b, c \in Up(a)$. If we assume that the cost of the equipments installed at node b is 200, then $p_{b1}^{k+1} = \frac{200+20}{10+20+10} = 5.5$ per unit of flow (note that the price charged by node a to node b is added to the equipment costs in the numerator). Node b will thus charge a price of $10 * 5.5 = 55$ to nodes $d, e \in Up(b)$ at iteration $k+1$. A price of $20 * 5.5 = 110$ is also associated with the local demand at node b. Once again, we have $55 + 55 + 110 = 200 + 20 = 220$. This procedure is repeated until all nodes have been processed.

3.3 Iterative heuristic

The complete heuristic is now described in the following, where k_{max} is a fixed number of iterations:

1. $k \leftarrow 0$
2. *changeflag*$\leftarrow 1$.
3. $p_{jt}^0 \leftarrow 0, \; j \in V, \; t \in T$.
4. While $(k \leq k_{max})$ and (*changeflag*$= 1$) do
   - 4.1 Downstream pass to produce a solution using $p_{jt}^k, j \in V, t \in T$.
   - 4.2 Upstream pass to produce $p_{jt}^{k+1}, \; j \in V, t \in T$.
   - 4.3 if $p_{jt}^k = p_{jt}^{k+1}, j \in V, t \in T$ then
     changeflag$\leftarrow 0$.

else

$p_{jt}^{k} \leftarrow p_{jt}^{k+1}, j \in V, t \in T.$

$k \leftarrow k+1.$

5. Return best solution found.

At each iteration, the heuristic restarts "from scratch" and takes new installation decisions at each node, but with different prices to guide the decision process. Note that the prices do not change as soon as two identical sets of prices are obtained for two consecutive iterations (i.e., as soon as two identical solutions are produced).

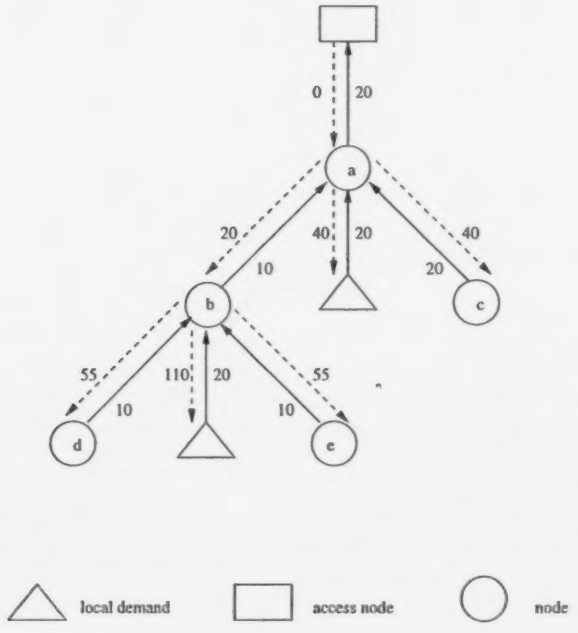

Figure 2: Flows and prices

3.4 A variant with price smoothing

A variant of this heuristic, obtained by "smoothing" the prices from one iteration to the next, can be described as follows.

1. $k \leftarrow 0$
2. *changeflag* $\leftarrow 1$
3. $p_{jt}^{0} \leftarrow 0,\ j \in V,\ t \in T.$
4. While $(k \leq k_{max})$ and (*changeflag*$= 1$) do

   4.1 Downstream pass to produce a solution, using $p_{jt}^{k},\ j \in V, t \in T.$

   4.2 Upstream pass to produce $p_{jt}^{k+1},\ j \in V, t \in T.$

4.3 if $max_{j \in V, t \in T} |p_{jt}^{k+1} - p_{jt}^{k}| < \epsilon$ then

changeflag $\leftarrow 0$.

else

$p_{jt}^{k+1} \leftarrow \alpha p_{jt}^{k+1} + (1-\alpha) p_{jt}^{k}$, $j \in V, t \in T$.

$k \leftarrow k+1$.

5. Return best solution found.

Basically, the prices generated by the upstream pass at iteration k (those to be used at iteration $k+1$) are combined with the current prices. The idea is to alleviate potential drastic changes from one iteration to the next by smoothing the values through a linear combination weighted by parameter α. Note that the stopping criterion now requires that modifications to the prices from one iteration to the next be less than or equal to some ϵ value. As opposed to the first heuristic (without smoothing), prices can change even when two consecutive identical solutions are generated.

4 Computational Results

In this section, computational results are reported on problems based on different cost structures and network sizes. The code was written in C++ and the tests were run on a 1.2GHz Sun workstation. We first describe the problem instances before reporting the results.

4.1 Problem Instances

4.1.1 Network

We have considered trees with 16 nodes (depths of 2 and 3), 90 nodes (depth 6) and 110 nodes (depth 9). For each network size, three different instances were generated: one is a balanced tree where all nodes, except the root and the leaves, have the same number of children; in the two other types, the number of children increases (respectively, decreases) with depth.

4.1.2 Horizon

The planning horizon extends over $|T| = 4$ time periods.

4.1.3 Equipments

There are three different types of concentrators with compression factors of .8, .4 and .2, and four different types of cables with capacity 1, 4, 16 and 64. Their costs are modulated along three dimensions: economies of scale, obsolescence and concentrator/cable cost ratio, as it is explained below.

Economies of scale. Economies of scale are associated with increasing concentrator compression factors and cable capacities. Let us assume that cables of capacity 1, 4, 16 and 64 are associated with types q=1, 2, 3 and 4. Similarly, concentrators with compression factors 0.8, 0.4 and 0.2 are associated with types l=1, 2 and 3, respectively. At period 1, the cost of a cable of type $q+1$ is derived from the cost of a cable of type q through the following formula:

$$c_{(q+1)ij1} = \lceil r * c_{qij1} \rceil, \ (i,j) \in A,$$

where $\lceil x \rceil$ is the ceiling of x and r is a random number chosen in the interval [1,2[, [2,3[or [3,4[depending if *high*, *medium* or *low* economies of scale are considered. In the *medium* case, the cost of a cable of type $q+1$ is thus between two and three times the cost of a cable of type q, even if its capacity is four fold. For concentrators (always at period 1), the formula is:

$$f_{(l+1)i1} = \lceil r' * f_{li1} \rceil, i \in V,$$

where r' is a random multiplier in the interval [1, 1.3[, [1.3, 1.7[, or [1.7, 2.0[depending if *high*, *medium* or *low* economies of scale are considered. The costs for the other periods are derived from the obsolescence patterns described below.

Obsolescence. The cost of any given equipment type decreases from one period to the next over the planning horizon. Six different obsolescence patterns have been considered for the cables and concentrators, as shown in Table 1. The pattern 1-1-1 indicates a fixed obsolescence cost of 10% from one period to the next. The pattern 1-2-3 indicates a more significant cost decrease in the later periods, namely a reduction of 30% from period 3 to period 4, as compared to only 10% from period 1 to period 2. Conversely, the pattern 3-2-1 indicates a more significant cost decrease in the earlier periods. The patterns 2-2-2, 1-2-6 and 6-2-1 are interpreted similarly.

Pattern	Period 1→Period 2	Period 2→Period 3	Period 3→Period 4
1-1-1	10%	10%	10%
1-2-3	10%	20%	30%
1-2-6	10%	20%	60%
2-2-2	20%	20%	20%
3-2-1	30%	20%	10%
6-2-1	60%	20%	10%

Table 1: Obsolescence in percent from one period to the next

Concentrator versus cable costs. Three different concentrator/cable cost ratios have been considered: *high*, where concentrator costs are relatively high with regard to cable costs and lead to solutions with only a few concentrators; *low* where concentrator costs are relatively low with regard to cable costs and lead to solutions with more concentrators; and *medium* where concentrator costs are between *low* and *high*. Examples of cost structures with these three ratios for obsolescence pattern 1-2-3 and *medium* economies of scale are shown in Tables 2, 3 and 4. The cost structures associated with the other obsolescence patterns are derived similarly from the "starting" costs in period 1.

Finally, for the cables, we have set $c_{qijt} = c_{qt}, \forall (i,j) \in A$. Hence, these costs are independent of the particular arc on which the cables are installed.

4.1.4 Demand

The demand always increases over the planning horizon. Two different patterns have been considered: *low* demand (increase of 20% from one period to the next) and *high* demand (increase of 100% from one period to the next).

4.2 Experimental setting

With 9 different trees, 54 cost structures (6 obsolescence patterns; 3 concentrator/cable cost ratios; 3 economies of scale) and 2 demand patterns, a total of 972 problem instances are available. Our algorithm was applied on these instances, using three different smoothing approaches by setting α to 1

cable capacity	*Period*			
	1	2	3	4
1	11	10	8	6
4	28	25	20	14
16	80	72	58	40
64	202	182	146	102
concentrator flow compression	*Period*			
	1	2	3	4
0.8	150	135	108	75
0.4	188	170	136	95
0.2	300	270	216	150

Table 2: Costs with obsolescence pattern 1-2-3, medium economies of scale and low concentrator/cable cost ratio

cable capacity	*Period*			
	1	2	3	4
1	11	10	8	6
4	28	25	20	14
16	80	72	58	40
64	202	182	146	102
concentrator flow compression	*Period*			
	1	2	3	4
0.8	200	180	144	100
0.4	260	234	188	130
0.2	422	380	304	212

Table 3: Costs with obsolescence pattern 1-2-3, medium economies of scale and medium concentrator/cable cost ratio

cable capacity	*Period*			
	1	2	3	4
1	11	10	8	6
4	28	25	20	14
16	80	72	58	40
64	202	182	146	102
concentrator flow compression	*Period*			
	1	2	3	4
0.8	300	270	216	152
0.4	386	348	278	195
0.2	562	505	404	282

Table 4: Costs with obsolescence pattern 1-2-3, medium economies of scale and high concentrator/cable cost ratio

(no smoothing), 0.5 and 0.1. The maximum number of iterations k_{max} was set to 50 in all experiments. The latter value was large enough to observe convergence on every problem instance.

In the first set of experiments, every tree topology was considered, but the cost and demand structures were fixed by considering only obsolescence pattern 1-2-3, *medium* concentrator/cable cost ratio, *medium*

economies of scale and *high* demand increase. These experiments were performed to see the impact of network size on algorithmic behavior, in particular on computation times. In the second set of experiments, an exhaustive experimentation was performed over all cost and demand structures, but on the largest problems only (i.e., trees with 110 nodes).

4.2.1 First set of experiments

The results obtained with obsolescence cost pattern 1-2-3, *medium* concentrator/cable cost, *medium* economies of scale and *high* demand increase are shown in Tables 5, 6 and 7, using smoothing values $\alpha = 1$, 0.5 and 0.1, respectively. Each entry is an average over the 3 trees associated with each network size. The first value is the improvement, in percentage, of the best solution found over the initial solution (the latter being obtained with prices set to 0); the second value is the iteration number associated with the best solution; the third value is the CPU time to reach the best solution; and the fourth value is the total CPU time.

Network size		
$\|V\| = 16$	$\|V\| = 90$	$\|V\| = 110$
0.1%	0.2%	0.7%
1	2	2
0.1s	0.5s	0.6s
2.2s	12.0s	14.2s

Table 5: Numerical results with $\alpha = 1$

Network size		
$\|V\| = 16$	$\|V\| = 90$	$\|V\| = 110$
0.1%	0.3%	0.9%
1	2	3
0.1s	0.5s	1.0s
2.2s	12.0s	15.1s

Table 6: Numerical results with $\alpha = 0.5$

Network size		
$\|V\| = 16$	$\|V\| = 90$	$\|V\| = 110$
0.1%	0.2%	0.9%
1	3	6
0.1s	0.7s	1.8s
2.2s	12.0s	15.1s

Table 7: Results with $\alpha = 0.1$

We observe an improvement of only 0.1% on the smallest instances with 16 vertices, versus an improvement close to 1% on the largest instances with 110 vertices. The best solutions are obtained within the first 10 iterations in all cases. This number tends to grow when problem size increases and when smaller α values are used. That is, smoothing allows the method to converge in a more "gradual" way towards its best solution, as illustrated in Figure 3 on a typical example. In this figure, the vertical axis corresponds to the solution value and the horizontal axis to the iteration number. Clearly, different α values lead to different search patterns. In this example, the first two solutions are the same but, afterwards, the evolution is more gradual when smaller α values are used. Although the general trend

is towards an improvement to the initial solution, the evolution from one iteration to the next is not monotonic, as uphill moves are observed. The computation times increase linearly with the number of vertices in the network. This is not surprising, given that the optimization is performed locally at each vertex.

The following section will now show how variations in demand patterns or equipment costs impact the performance of the method.

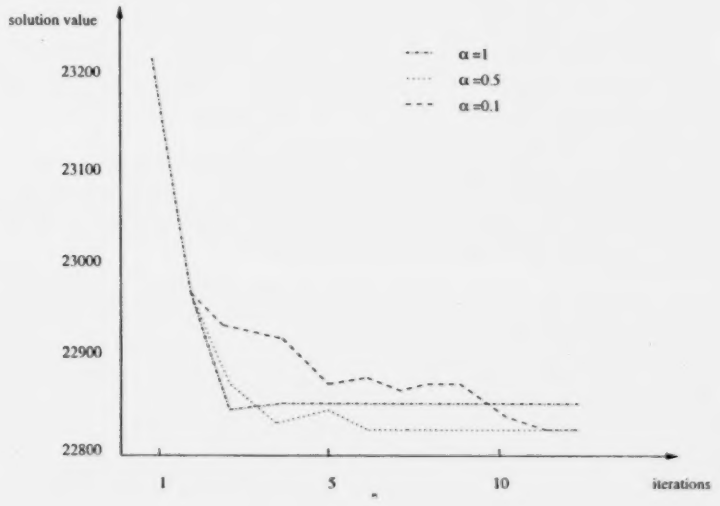

Figure 3: Solution evolution with different α values

4.2.2 Second set of experiments

The following tables report the results obtained with different cost and demand structures on the largest trees with 110 nodes. Each entry is an average over 3 trees. The first value is the improvement, in percentage, of the best solution over the initial solution and the second value is the iteration number associated with the best solution. The CPU times are not shown, as these values are very similar to those in Tables 5, 6, and 7, where each iteration takes approximately 0.3 second for $|V| = 110$.

Although the number of iterations needed to get the best solution equals 28 in one experiment, this number is under 10 in most cases. We can see that larger improvements are observed, with some that are now close to 10%. This is true, in particular, when the concentrator/cable cost ratio and economies of scale are *high*. In this situation, there are fewer concentrators in a solution, and each modification to their configuration (i.e., location, type) significantly impacts the solution value. When the cost ratio is *high*, there are approximately 10 to 15 concentrators in a solution, as opposed to 40 to 50 concentrators when this ratio is *low*. Small improvements, typically below 1%, are observed when the concentrator/cable cost ratio is *medium* or *low* (except with *high* economies of scale). On the other hand, the method seems to be robust with regard to economies of scale.

Larger improvements are generally observed with *low* demand increase, although the final outcome depends on the particular cost structure (see, for example, the results with *medium* cost ratio and *high* economies of scale). As in the first set of experiments, smaller α values lead to more gradual improvements to the current solution (as indicated by the larger number of iterations), but the solution obtained at the end is not necessarily of better quality.

concentrator/cable cost ratio	*economies of scale*	*obsolescence cost pattern*					
		1-1-1	1-2-3	1-2-6	2-2-2	3-2-1	6-2-1
high	high	9.2% 2	8.0% 2	7.1% 2	7.3% 2	6.7% 2	6.8% 3
	medium	4.4% 3	3.4% 2	4.1% 2	2.6% 2	3.2% 2	1.9% 1
	low	4.5% 2	4.4% 3	4.3% 2	3.8% 2	3.5% 2	4.1% 2
medium	high	2.5% 1	3.3% 2	2.5% 2	2.6% 2	4.2% 2	4.9% 2
	medium	0.0% 0	0.1% 1	0.3% 1	1.0% 2	1.0% 2	0.7% 1
	low	1.1% 3	0.9% 2	0.5% 1	0.6% 2	0.3% 1	0.8% 2
low	high	3.1% 2	2.9% 2	2.5% 2	2.4% 2	2.4% 2	5.5% 4
	medium	0.2% 1	0.1% 1	0.6% 1	0.0% 0	0.3% 1	0.9% 2
	low	1.2% 1	0.8% 1	0.7% 1	0.5% 1	0.1% 1	0.7% 2

Table 8: Results with $\alpha = 1$ and low demand increase

concentrator/cable cost ratio	*economies of scale*	*obsolescence cost pattern*					
		1-1-1	1-2-3	1-2-6	2-2-2	3-2-1	6-2-1
high	high	3.5% 2	5.4% 3	4.1% 2	3.4% 2	4.0% 3	5.5% 4
	medium	0.3% 2	0.2% 1	0.8% 2	0.0% 0	0.1% 1	0.0% 0
	low	0.3% 1	0.0% 0	0.3% 1	0.0% 0	0.1% 1	0.3% 3
medium	high	3.6% 3	4.1% 2	4.4% 3	2.5% 2	4.3% 3	5.6% 5
	medium	0.0% 0	0.7% 2	0.5% 2	0.6% 2	0.1% 2	0.0% 0
	low	0.7% 1	1.5% 1	1.5% 2	0.2% 1	0.3% 1	0.8% 1
low	high	1.1% 2	2.1% 2	3.4% 3	0.2% 1	0.0% 0	1.5% 3
	medium	0.9% 2	1.3% 2	0.9% 1	0.5% 1	0.0% 0	0.0% 0
	low	0.0% 0	0.0% 0	0.0% 0	0.0% 0	0.1% 1	0.0% 0

Table 9: Results with $\alpha = 1$ and high demand increase

concentrator/cable cost ratio	*economies of scale*	*obsolescence cost pattern*					
		1-1-1	1-2-3	1-2-6	2-2-2	3-2-1	6-2-1
high	high	8.9% 3	8.0% 2	7.1% 2	7.3% 2	6.7% 3	6.8% 4
	medium	4.6% 5	3.5% 3	4.1% 2	2.8% 3	2.9% 3	1.9% 1
	low	4.7% 3	4.7% 3	4.6% 2	4.5% 3	3.9% 3	5.4% 7
medium	high	2.5% 2	3.3% 2	2.5% 2	2.6% 2	4.4% 2	5.2% 2
	medium	0.0% 0	0.1% 1	0.3% 1	1.0% 2	1.0% 2	0.8% 2
	low	1.2% 4	1.1% 2	0.6% 2	0.8% 3	0.3% 1	0.6% 3
low	high	3.1% 5	2.9% 2	2.5% 2	2.4% 2	2.4% 2	6.5% 6
	medium	0.2% 1	0.1% 1	0.6% 1	0.0% 0	0.4% 3	1.0% 3
	low	1.2% 1	0.8% 1	0.7% 1	0.5% 1	0.1% 1	0.7% 2

Table 10: Results with $\alpha = 0.5$ and low demand increase

concentrator/cable cost ratio	*economies of scale*	*obsolescence cost pattern*					
		1-1-1	1-2-3	1-2-6	2-2-2	3-2-1	6-2-1
high	high	3.6% 2	5.4% 2	4.1% 2	3.6% 5	4.6% 17	5.6% 16
	medium	0.3% 3	0.3% 2	1.0% 4	0.0% 0	0.2% 1	0.0% 0
	low	0.4% 2	0.0% 0	0.4% 2	0.0% 0	0.1% 1	0.5% 4
medium	high	3.9% 4	4.1% 2	4.4% 3	2.5% 3	4.1% 6	6.0% 18
	medium	0.0% 0	0.9% 3	0.7% 3	0.7% 2	0.3% 2	0.0% 0
	low	0.7% 1	1.5% 1	1.7% 3	0.3% 2	0.3% 1	0.8% 2
low	high	1.1% 2	2.6% 3	3.4% 3	0.3% 3	0.0% 0	1.5% 3
	medium	0.9% 2	1.6% 4	0.9% 1	0.5% 1	0.0% 0	0.0% 0
	low	0.0% 0	0.0% 0	0.0% 0	0.0% 0	0.1% 1	0.0% 0

Table 11: Results with $\alpha = 0.5$ and high demand increase

concentrator/cable cost ratio	*economies of scale*	*obsolescence cost pattern*					
		1-1-1	1-2-3	1-2-6	2-2-2	3-2-1	6-2-1
high	high	8.9%	8.0%	7.1%	7.3%	6.7%	6.8%
		3	2	2	2	3	4
	medium	4.2%	3.4%	3.8%	2.7%	3.0%	1.9%
		6	4	5	4	3	3
	low	4.8%	4.7%	4.9%	4.3%	3.8%	5.5%
		7	5	4	8	2	28
medium	high	2.5%	3.3%	2.5%	2.6%	4.2%	4.9%
		5	4	4	4	5	2
	medium	0.0%	0.2%	0.3%	1.0%	1.1%	0.8%
		0	4	1	4	8	2
	low	1.5%	1.2%	0.8%	0.7%	0.3%	1.0%
		12	2	3	3	2	11
low	high	3.1%	2.9%	2.5%	2.4%	2.4%	6.5%
		3	2	2	2	2	11
	medium	0.3%	0.2%	0.9%	0.0%	0.5%	1.1%
		5	3	2	0	2	10
	low	1.2%	0.8%	0.7%	0.5%	0.1%	0.8%
		1	1	1	1	4	3

Table 12: Results with $\alpha = 0.1$ and low demand increase

concentrator/cable cost ratio	*economies of scale*	*obsolescence cost pattern*					
		1-1-1	1-2-3	1-2-6	2-2-2	3-2-1	6-2-1
high	high	3.6%	5.4%	4.1%	3.4%	3.6%	5.0%
		3	3	2	2	5	13
	medium	0.3%	0.3%	0.9%	0.0%	0.2%	0.0%
		6	7	15	0	1	0
	low	0.5%	0.0%	0.4%	0.0%	0.1%	0.7%
		4	0	2	0	1	6
medium	high	3.9%	4.1%	4.9%	2.5%	4.3%	5.5%
		15	5	6	9	18	16
	medium	0.4%	0.9%	0.9%	0.9%	0.4%	0.0%
		2	6	5	3	2	0
	low	0.9%	1.5%	1.7%	0.5%	0.3%	1.0%
		3	2	8	4	1	3
low	high	1.1%	2.1%	3.4%	0.2%	0.0%	1.8%
		4	13	7	6	0	16
	medium	0.9%	1.3%	0.9%	0.5%	0.1%	0.1%
		2	1	1	1	0	6
	low	0.0%	0.0%	0.0%	0.0%	0.1%	0.0%
		0	0	0	0	1	0

Table 13: Results with $\alpha = 0.1$ and high demand increase

5 Conclusion

In this paper, we have presented an iterative heuristic that solves a local capacity expansion problem at each node of a telecommunications network with a tree topology. Some global information is avail-

able through prices that are propagated in the network and used to guide the decision process. The computational results show that substantial improvements over the initial solution (produced with null prices) can be obtained. Futhermore, the use of a smoothing parameter that preserves some information about the search history, allows the heuristic to follow different search patterns in the solution space. Assuming that the reported heuristic is a single component in a more elaborate global search procedure, this would allow the creation of a pool of different solutions (which could be used to restart, diversify or intensify the search).

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